RELATION BETWEEN MATHEMATICAL REASONING ABILITY AND NATIONAL FORMAL DEMANDS IN PHYSICS COURSES

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It is widely accepted that mathematical competence is of great importance when learning physics. This paper focuses on one aspects of mathematical competence, namely mathematical reasoning, and how this competency influences students' success in physics. Mathematical reasoning required to solve tasks in physics tests, within a national testing system, is separated into imitative and creative mathematical reasoning. The results show that students lacking the ability to reason creatively are more likely not to do well on national physics test, thus not fully mastering the physics curricula. It is further discussed how the high demands of creative mathematical reasoning in physics tests stand in contrast to what is known about the educational practices in mathematics and physics in upper secondary school.

INTRODUCTION

Many scholars discuss the importance of understanding how mathematics is used in physics and how students' mathematical knowledge affects their learning of physics, e.g., Basson (2002) who mentions how difficulties in learning physics not only stem from the complexity of the subject but also from insufficient mathematical knowledge, Bing (2008), in his discussion of the importance of learning the language of mathematics when studying physics, as well as Redish and Gupta (2009), who emphasise the need to understand the cognitive thinking of experts in order to teach mathematics for physics more effectively to students.

According to the Swedish National Agency for Education (2009a) a common activity in physics classes is students using physics laws and formulas to solve routine tasks. The most common homework is to read in the textbook and/or to solve various tasks posed in the book, and sometimes to memorise formulas and procedures (ibid.). Similar results are described by Doorman and Gravemeijer (2009), who notice that most of the attention in both physics and mathematics in school is paid to the manipulations of formulas instead of focusing on why the formulas work. Redish (2003) states that practice, in the meaning that students just solve various tasks, is necessary but not enough to develop a deeper understanding of the underlying physics concepts. Students must learn both *how* to use the knowledge and *when* to use it.

The impact of mathematical reasoning *on mathematical* learning has been discussed and studied from multiple perspectives. Schoenfeld (1992), for example, points out that a focus on rote mechanical skills leads to poor performance in problem solving in contrast to the performance of mathematically powerful students. Lesh and

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Zawojeskij (2007) discuss how emphasising low-level skills does not give the students the abilities needed for mathematical modelling or problem solving, neither to draw upon interdisciplinary knowledge. Students lacking the ability to use creative mathematical reasoning thus get stuck when confronted with novel situations, and this negatively influences their possibilities to learn (Lithner, 2008). Since mathematics is a natural part of physics, it is reasonable to assume that the ability to use mathematical reasoning is an integral part of the physics knowledge students are assumed to achieve in physics courses.

FRAMEWORK

During studies on how students engage in various kinds of mathematical activities, Lithner (2008) developed a framework for characterising students' mathematical reasoning. The framework distinguishes between *creative mathematical founded reasoning* (CR) and *imitative reasoning* (IR). To be regarded as CR the following criteria should be fulfilled: **i. Novelty**. A new reasoning sequence is created or a forgotten one is recreated. **ii. Plausibility**. There are arguments supporting the strategy choice and/or strategy implementation motivating why the conclusions are true or plausible. **iii. Mathematical foundation**. The arguments made during the reasoning process are anchored in the intrinsic mathematical properties of the components involved in the reasoning (Lithner, 2008, p. 266).

Reasoning categorised as IR fulfils: **i.** The strategy choice is founded on recalling a complete answer. **ii.** The strategy implementation consists only of writing it down (Lithner, 2008, p. 258), or **i.** The strategy choice is to recall a solution algorithm. The predicted argumentation may be of different kind, but there is no need to create a new solution. **ii.** The remaining parts of the strategy implementation are trivial for the reasoned, only a careless mistake can lead to failure (ibid. p. 259).

In the application of the framework for the analyses described in this paper, an additional category, defined in Johansson (2103), is used. This category consists of those tasks that can be solved by only using physics knowledge; and this category is called *non-mathematical reasoning* (NMR). Physics knowledge is here referred to as relations and facts that are discussed in the physics courses and not in the courses for mathematics, according to the syllabuses and textbooks, e.g. that angle of incidence equals angle of reflection.

RESEARCH QUESTIONS

There is a significant amount of educational research on the relation between the school subjects of mathematics and physics that support the necessity of different mathematical competencies when learning physics. However, no studies on what type of mathematical reasoning is required of physics students were found. As an approach to the assumption that students' ability to reason mathematically affects how they master the physics curricula, this study use a previous analysis (Johansson, 2013) of the mathematical reasoning requirements to solve tasks in physics tests together with actual students' results on the same tests.

The Swedish national physics tests are the government's way of concretising the physics curricula. Thus, the requirements of mathematical reasoning to solve tasks in national physics tests should capture the mathematical reasoning that is required to master or fully master the curricula. The goals and the subject descriptions in the Swedish curricula of what it means to know physics are quite rich and are highly in accordance with the content and cognitive domains in the TIMSS Assessment framework (Garden et al. 2006; Swedish National Agency for Education, 2009b). This alignment with TIMSS suggests that the results from this study are relevant to an international context.

By addressing the questions: *Is it possible for a student to get one of the higher grades, Pass with distinction and Pass with special distinction, without using CR?*, and *If it is possible, how common is it?*, this study examines how the universal requirement of a mathematical reasoning competency to master the physics curricula relates to a specific assessment system's formal demands, in this case Sweden's.

METHOD

The empirical data consisted of student data from eight randomly chosen Swedish national physics tests for upper secondary school, and the tasks in the tests. There are mainly two different physics courses in the Swedish upper secondary school. Physics A that is compulsory for all natural science and technology students and Physics B that is an optional continuation. The tasks had previously been categorised according to mathematical reasoning requirements (Johansson, 2013), and together the tests comprised 169 tasks. The tests, which are classified to not authorised users, and the student data were used by permission from Department of Applied Educational Science at Umeå University, the department in charge of the National Test Bank in Physics. Student data come as excel sheets, one sheet for each test. The sheets contain information about individual students' grade, whereas the grade is one of the following: *Not Pass* (IG), *Pass* (G), *Pass with distinction* (VG), and *Pass with special distinction* (MVG). Further information in the sheets are individual student's scores on each task separated in G- and VG-scores, and their total score on the tests. No names of the students are present in the sheets, instead each student has got an ID-number. The IDs are unidentifiable for anyone outside the Department of Applied Educational Science at Umeå University, so data could be considered anonymous. The number of student data for each test varies from 996 to 3666.

For each test there are certain score levels the students need to attain to get a certain grade. To get the grade MVG, students need to fulfil certain quality aspects besides the particular score level. To decide if it is possible for a student to get one of the higher grades, VG or MVG, without using any kind of CR, each test was first analysed separately. This analysis consisted in comparing the score level for each grade with the maximum scores that are possible to obtain, given that the student only has solved (partly or fully) IR- and/or NMR- tasks. The available student data did not give any information about which of the qualitative aspects required for MVG the students have fulfilled, but the data sheets included students grades, thus MVG could be included in the analyses as one of the higher grades. After analysing if it is possible at all to receive the grades VG or MVG without solving any CR-tasks, students' actual results on the categorised tasks for those particular tests are summed up. The proportion of students who only got scores from IR- and/or NMR-tasks is then graphed with respect to the different grades.

RESULTS

Table 1 shows how the scores, possible to obtain on each of the eight tests that were analysed, are distributed among the reasoning categories IR and NMR. The table also includes the levels for the grades G, VG and MVG. The notation for the scores follows the convention G/VG.

Test	Max score (G/VG)	Min required score for G	Min required score for VG	Min required score for MVG	Max scores for IR-tasks	Max scores for NMR-tasks	Max score possible without CR-tasks
Physics A May 02	43 (26/17)	12	25 (with at least 6 VG scores)	25 (with at least 12 VG scores)	12/0	3/3	18 (with 3 VG)
Physics A Dec 04	40 (23/17)	12	24 (with at least 5 VG scores)	24 (with at least 12 VG scores)	14/3	3/3	23 (with 6 VG)
Physics A May 05	38 (22/16)	12	24 (with at least 6 VG scores)	24 (with at least 12 VG scores)	12/3	8/4	27 (with 7 VG)
Physics B May 02	48 (23/25)	12	27 (with at least 7 VG scores)	27 (with at least 13 VG scores)	11/4	2/0	17 (with 4 VG)
Physics B May 03	43 (23/20)	12	25 (with at least 6 VG scores)	25 (with at least 13 VG scores)	12/8	5/1	26 (with 9 VG)
Physics B May 05	44 (22/22)	12	25 (with at least 6 VG scores)	25 (with at least 12 VG scores)	8/5	7/2	22 (with 7 VG)
Physics B Feb 06	43 (22/21)	12	25 (with at least 7 VG scores)	25 (with at least 13 VG scores)	11/7	9/9	36 (with 16 VG)
Physics B April 10	44 (24/20)	12	25 (with at least 6 VG scores)	25 (with at least 12 VG scores)	9/4	4/1	18 (with 5 VG)

Table 1: Analysis of the distribution of G- and VG-scores among IR- and NMR-tasks.

For example, for the Physics A test from May 02 is the maximum score 43, and of these scores are 26 G-scores and 17 VG-scores. To pass this particular test a student has to have at least 12 scores, it does not matter if these scores are G- or VG-scores. To get the higher grade VG, a student has to have at least 25 scores and at least 6 of these scores have to be VG-scores. To get the highest grade, MVG, a student has to have at least 25 scores and at least 12 scores of these have to be VG-scores. As mentioned above, students also have to fulfil some additional quality aspects to achieve the grade MVG. Further, for the Physics A test from May 02, if a student only solves all tasks categorised as IR, he/she can obtain at most 12 G scores. If a student only solves all tasks categorised as NMR, he/she can obtain 3 G-scores and 3 VG-scores. Solving all IR- and NMR-tasks thus result in total 18 scores of which 3 are VG-scores. The scores for the rest of the analysed tests are presented in the same way.

In three of the eight tests (highlighted in Table 1) it is possible to get the grade VG by solving tasks not requiring any CR. In one of these tests, Physics B from February 2006, it is with respect to score level possible to obtain the grade MVG by solving only IR- and NMR-tasks. The analysis does not reveal anything about whether the requirements of the qualitative aspects for MVG are possible to fulfil by solving only these kinds of tasks.

Figure 1 illustrates the proportion of students on the three highlighted tests in Table 1 who only had solved IR- and/or NMR-tasks graphed with respect to their grades on the tests.

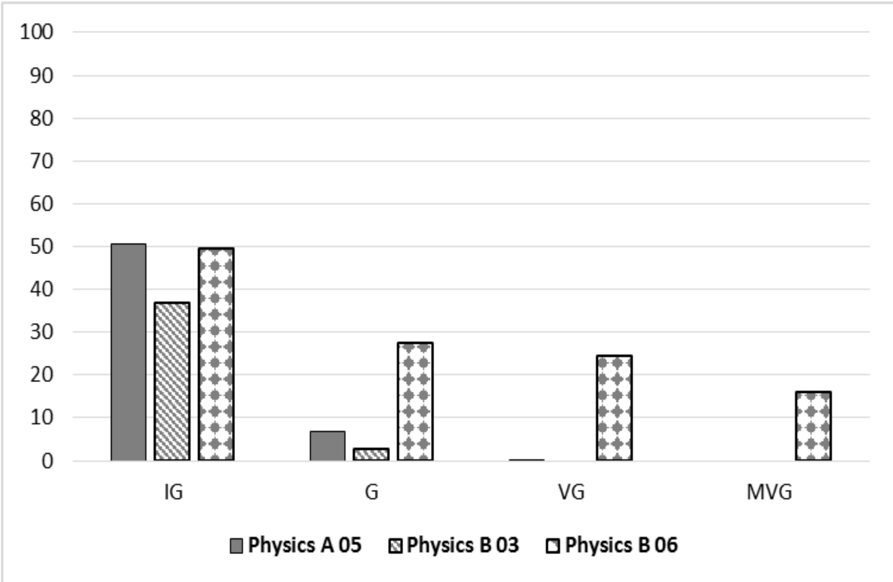

Figure 1: Proportion of students who only solved IR- and/or NMR-tasks with respect to the different grades.

It turned out that it is not frequently occurring that a student gets a higher grade than G by only solving these kinds of tasks. In the test for Physics A from 2005, only 0.17 % of the students got a higher grade; and in the Physics B test from 2003 none of the students got higher grades than G. The Physics B test from 2006 seems to be an exception though, since 25% of the students taking this test got a VG and 17% got a MVG. The analysis of how the scores are distributed among the reasoning categories for the different tests shows that the Physics B test from 2006 contains a lot more scores in the NMR category than any of the other tests (see Table 1). The total scores possible to obtain by only solving NMR-tasks are 18; nine of these are VG-scores, which is more than enough to fulfil the requirement for a VG (minimum 7 VG).

DISCUSSION

The analysis shows that it is possible to receive a higher grade than G by using only IR and NMR on three out of eight tests. When this result is compared with student data it is revealed that not using any CR, still receiving a higher grade, only occurs on one of the eight tests. This particular test, for which this occurs, is slightly different compared to the other tests with respect to how the scores are distributed among the reasoning categories (see Table 1). Further analysis of the test shows that tasks where it is possible to show the qualitative aspects required for the highest grade can be solved without using any mathematical reasoning i.e. these tasks are in the NMR category. This explains the higher frequency of students receiving the higher grades by using only IR and NMR, compared to the other tests.

The analysis of the tests furthermore shows that it is impossible to pass six of the eight tests without solving any tasks requiring mathematical reasoning. As seen in Table 1 it is only on the tests Physics A, May 05 and Physics B, Feb 06 a student can get at least the score 12, which is required to pass a test, by only solving NMR-tasks. These results strengthen the outcome from the author's previous study, which are that the ability to reason mathematically is an important competency and an integral part when taking physics tests (Johansson, 2013).

Mathematical reasoning is defined as a process to reach conclusions when solving tasks (Lithner, 2008). When students have the ability to use creative mathematical founded reasoning, they know how to argue and justify their conclusions and they can draw on previous knowledge. The result in the present study shows that CR is required to succeed on most of the physics tests. The alignment between the TIMSS framework and the Swedish policy documents suggests that this is a universal demand on upper secondary physics students. Viewing the physics tests from the National Test bank as an extension of the national curricula, one can assume that students' results on the tests are a measure of their knowledge in physics. It is well known that a focus on IR can explain some of the learning difficulties that students have in mathematics. The results above show that a focus on IR when studying physics in upper secondary school will make it hard for the students to do well on the physics tests, thus fully mastering the physics curricula. Therefore, a reasonable conclusion is that focusing on IR can hinder students' development of knowledge in physics, similar to results found about mathematics, and a creative mathematical reasoning competency can be regarded decisive.

The argumentative side of mathematics, which is a reasoning based on intrinsic properties of the components involved in the task-solving process, seems to be an inseparable part of mastering physics. All students should have the same possibilities to achieve the goals in the physics curricula. Therefore, they ought to be given the opportunity in school to develop and practice this creative mathematical reasoning competency that is required. As mentioned in the introduction, it is common in the physics classes that students solve routine tasks and focus on manipulations on formulas instead of focusing on the conceptual understanding of the underlying principles (Doorman & Gravemeijer, 2009; Swedish National Agency for Education, 2009a). Although it is the physics perspective that is discussed in the above studies, it is reasonable to assume that if there is more focus on physics procedures than on the understanding of physics concepts, there is also little focus on creative mathematical reasoning.

It is not only the physics classes that might provide students the opportunity to develop a mathematical reasoning competency, this competency is of course relevant also in the mathematics classes. According to studies about the learning environment in mathematics classes, the focus is on algorithmic procedures and the environment does not provide extensive opportunities to learn and practice different kinds of reasoning (e.g., Boesen, Lithner & Palm, 2010). During observations of classroom activities it was shown that opportunities to develop procedural competency was present in episodes corresponding to 79% of the observed time; compared to episodes involving opportunities to develop mathematical reasoning competency, which were present in 32% of the observed time (Boesen et al., 2014). Also tests have an indirect role for students learning, both as formative, when students get feedback on their solutions, and as summative, when the character of the tasks give students indications of what competences that are sufficient for handling mathematical tasks. Analyses of teacher-made mathematics tests have shown that these focused largely on imitative reasoning, in contrast to the national mathematics tests, which had a large proportion of tasks requiring creative mathematical reasoning (Palm, Boesen, & Lithner, 2011). Altogether, the above discussion shows that students are provided limited opportunities to develop the creative mathematical reasoning competency that is formally required to master the physics curricula. The importance of the relation between mathematics and physics has been known for a long time. The result from the present study, that the ability to creatively mathematically argue and reason is decisive in order to fully master the physics curricula, should have implications on how the education is organised and carried out.

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